**Order-Size (Q), Reorder-Point (R) model: A Brief Introduction**

It is time to expand our horizons and consider inventory-flow for items that are not limited to single-period but are replenished (from supplier or upstream warehouse) continuously as sales take place. Let us start with focusing on one single inventory-location and one item. There are two decisions that control the inventory flow of such an item – what should be the size of replenishment order (call it Q) and when should we place a replenishment order (reorder point R)?

***Costs and other performance measures***

We consider two costs: every time an order is made there is a transportation cost and there is a purchasing cost. We capture these with a combination of fixed cost and a variable per-unit cost. The part of the cost that is independent of the size of the order is called order cost S and the cost that is incurred per-unit is called p. For example, if transportation contractor charges a full truck cost (say $1000) irrespective of the order size and supplier charges a purchase price of $50 per unit then S=$1000 per order and p=$50 per unit.

We will also consider holding cost H as either given or as annual interest on p. The cost structure is exactly the same as EOQ model. Units of H is $ per unit per time-unit.

The other performance measure is the service level we provide to the customer. The time between the deliveries of two orders is a cycle. We will define service level as fraction of cycles in which we do not run out of stock (that is, we satisfy all demand in that cycle). We will work with a given *target service level*we want to achieve.

Recall that in EOQ model we never ran out because we considered demand as fixed and given. We will now consider demand as variable which means there is always a possibility of running out before the next replenishment comes in.

***Demand and Lead Time***

We will consider demand per period (a day or a week for example) to be normally distributed with a given mean and a given standard deviation.

The time between we place a replenishment order and its arrival from supplier is lead time. For now, consider it a fixed time (days or weeks).

***Order Size Q decision***

As you may think, order-size decision can be made just the way we solved the EOQ problem. Demand rate D is mean demand per time-unit. Order cost S and holding cost H have been defined above. We can use the EOQ formula to determine order-size Q. In practice, though, the order-size is often governed by practical considerations like truck-size and container-size. Usually, we will just consider order size Q as given.

Recall that order size Q determines *cycle stock Q/2*.

***Reorder Point R Decision***

Reorder point is defined as the level of inventory that will trigger the release of a replenishment order (of size Q) to the supplier. We define it as the level of inventory that is sufficient to meet all demand till the time supplier actually delivers the order. Remember that supplier will take a lead time to deliver the order. Therefore, at the reorder point, we should have enough units in inventory to meet *demand during lead time (DDLT).* Since demand is variable, use the following formulas to characterize demand during lead time.

DDLT mean = mean demand per time-unit \* lead time in time-units

DDLT std. dev. = (square root of lead time) \* std.dev. of demand

We set reorder point to achieve the target service level. Given a target service level we can compute z=Normsinv (service level) and then:

Reorder point R = DDLT mean + z\* DDLT std.dev.

We define *safety stock SS* as SS=z\*DDLT std.dev.

It is possible now to compute the total average inventory to provide a target service level. We carry cycle stock Q/2 and safety stock SS in order to provide a given target service level.

Finally, pipeline stock refers to the average inventory that is on road (or on rail track, ocean, air) but has not yet arrived in inventory. It turns out that average pipeline stock is just mean DDLT.

***Extensions***

In case lead time is variable, we should measure its mean and std.dev. and then compute DDLT std.dev. as following. Everything else remains the same. DDLT std.dev. =

Square root of [lead-time mean\* (std.dev. of demand)2 + (mean demand)2 \* (std.dev. of lead time)2 ]

Sometimes we have to think about demand across several products or several locations. For example, if there is one product that can substitute for another, we may have to think of the total demand across two products. Or if a warehouse is supplying a product to two retail locations then the product demand at warehouse is the sum of two retailers’ demand. When adding demand distributions like this, the rule is that you can add averages but you cannot add standard deviations. But variances (squared standard deviations) can be added. After adding variances, we can take a square root to get the standard deviation for the total demand.

***Summary of Formulas***

Cycle stock = order size Q/2

DDLT mean = mean demand per time-unit \* lead time in time-units

DDLT std. dev. = (square root of lead time) \* std.dev. of demand

Reorder point R = DDLT mean + z\* DDLT std.dev

Safety stock SS=z\*DDLT std.dev.

Pipeline stock = mean DDLT

**Order-Size (Q), Reorder-Point (R) model: Practice Problems**

**1.** An ophthalmologist’s office operates 52 weeks per year, 6 days a week. It purchases disposable contact lenses for $11.70 per pair and sells for $50. Demand is 90 pairs per week. Order cost is $54 per order. Annual interest rate is 27%. Lead-time is 3 weeks. Standard deviation of weekly demand is 15 pairs. Find a (Q,R) policy that gives 98% service-level. Clearly identify different types of stocks: cycle, safety and pipeline.

**2.** Fancy Paints is a small paint store. Fancy paints stocks 200 different SKUs (stock-keeping units) and places replenishment orders of the size 50 gallons. The order arrives four weeks later. To make it simple, let us assume that weekly demand for each SKU is the same and is normally distributed with mean 10 gallons and standard deviation 9 gallons. Fancy paint maintains a 95% service level.

(a) What is the safety stock at the store?

(b) Fancy Paints purchases a color-mixing machine. It allows Fancy Paints to store only a “base” liquid and then use small color tablets to quickly create any of the above 200 SKUs. This requires carrying inventory of color tablets but they are inexpensive and easy to stock in very large quantities. Ignoring tablets for now, how much average inventory of “base” liquid should be kept at store? Lead time and service level remains the same.

**Order-Size (Q), Reorder-Point (R) model: Practice Problems Solutions**

**1.** An ophthalmologist’s office operates 52 weeks per year, 6 days a week. It purchases disposable contact lenses for $11.70 per pair and sells for $50. Demand is 90 pairs per week. Order cost is $54 per order. Annual interest rate is 27%. Lead-time is 3 weeks. Standard deviation of weekly demand is 15 pairs. Find a (Q,R) policy that gives 98% service-level. Clearly identify different types of stocks: cycle, safety and pipeline.

*Order Size Q*

In some problems, Q is given. Here it is not, so let us use Q=EOQ.

set Q = EOQ = =400

ordering cost = (90\*52/400)\*54=631.80 per year

*Reorder Point R*

Demand during lead-time: mean = 90\*3 = 270 ; std. dev. = sqrt(3)\*15 = 26

service level =98%, corresponding z = 2.05,

R = 270+2.05\*26 = 323.3

SS = 2.05\*26=53.3

Average inventory = Cycle stock +SS = (400/2)+ 53.3=253.3

Holding cost = 0.27\*11.70\*253.3 = 800.17 per year

Cycle stock is 200, safety stock is 53.3 and pipeline stock is equal to mean DDLT=270.

**2.** Fancy Paints is a small paint store. Fancy paints stocks 200 different SKUs (stock-keeping units) and places replenishment orders of the size 50 gallons. The order arrives four weeks later. To make it simple, let us assume that weekly demand for each SKU is the same and is normally distributed with mean 10 gallons and standard deviation 9 gallons. Fancy paint maintains a 95% service level.

(a) What is the safety stock at the store?

For each SKU:

DDLT mean is 10\*4=40. DDLT std.dev. =18.

95% service level gives z =1.64. Safety stock = 1.64\*18=29.52

Across 200 SKU, total safety stock at the store = 200\*29.52=5904

(b) Fancy Paints purchases a color-mixing machine. It allows Fancy Paints to store only a “base” liquid and then use small color tablets to quickly create any of the above 200 SKUs. This requires carrying inventory of color tablets but they are inexpensive and easy to stock in very large quantities. Ignoring tablets for now, how much average inventory of “base” liquid should be kept at store? Lead time and service level remains the same.

Weekly demand for base liquid:

mean = 200\*10=2000;

standard deviation = square root of (200\*92) = 127.28

(Remember that variances (std.dev.2) can be added. We get this by adding variances 92 across 200 SKUs and then taking square root to getback to std.dev.)

DDLT for base liquid: mean = 2000\*4=8000; std.dev.= =254.56

Safety stock for base liquid = 1.64\*254.56=417.48